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Exploratory Study of a Self-Regulation Mathematical Writing Strategy: Proof-of-Concept

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Content writing contributes to students' positive learning and outcomes. Mathematical-content writing has been shown to positively support students' mathematical reasoning, but it is challenging for many students with and without learning disabilities. This study explores proof-of-concept evaluation to extend self-regulated strategy development, an evidence-based practice in writing, to mathematical content-specific writing. Twenty-seven students participated in this pilot study, which included seven students with disabilities. The intervention consisted of six lessons that taught students a strategy to self-regulate written responses to open-response mathematics problems. Results indicate that mathematics fact fluency correlated with mathematical writing quality and demonstrated to be a potential predictor for mathematical writing performance. Mean scores indicated that students in the treatment group outperformed students in the control group, although the difference was not statistically significant. Results support the proof-of-concept that teaching students a self-regulated strategy for mathematical writing may improve student outcomes, although future research is needed with a greater number of participants that receive longer dosage of the intervention to more confidently support this claim. The implications for future research and practice were discussed.

Keywords: **mathematics, writing, self-regulated strategy development (SRSD), mathematical writing, reasoning, elementary, middle school.**

INTRODUCTION

By the time students enter middle-grades education in the US (i.e., Grades 5-8), they are expected to have developed a foundation of conceptual and procedural knowledge regarding pre-algebraic skills. Educators have expressed concerns that students often enter middle school un- or underprepared for success in mathematics courses. Evidence of this poor achievement may also be seen in eighth-grade students' performance scores on high-stakes mathematics assessments (e.g., National Assessment of Educational Progress (NAEP; National Center for Educational Statistics, 2019) and is even more abysmal for students with disabilities. According to NAEP (2019), mathematics scores of eighth-grade students with disabilities averaged below the basic level. This result is not surprising when considering lifelong chal-

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lenges students with disabilities experience with mathematics. One potential way to improve mathematics achievement, reasoning, and conceptual understanding is to engage students in mathematical-content writing.

Written expression in mathematics is understood as an essential element in facilitating mathematical concepts and reasoning (The National Council of Teachers of Mathematics [NCTM], 2000). The NCTM (2010) has highlighted the importance of written communication in mathematics classrooms because it encourages students to reach conclusions with logical sequences by using mathematical language and conceptual ideas (Hebert & Powell, 2016). The Common Core State Standards for Mathematics (CCSSM) has also stated that students are expected to "construct viable arguments and critique the reasoning of others" (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010, p. 6). In efforts to evaluate these important skills, many states in the United States require students to respond to open-ended questions by explaining their reasoning process in written language as early as third grade. Although the importance of written expressions of mathematics is being emphasized and recommended as an important element of mathematical proficiency, the concept of mathematical writing (MW) spans two different academic domains, writing and mathematics, both of which pose learning challenges to students with and without learning disabilities (LD). In this paper, we first discuss benefits of content-writing in mathematics and share a current review of the literature. Next, we discuss challenges students with LD may encounter. Finally, we present our intervention as a way to address students' learning challenges to improve written expression of mathematical reasoning.

Writing as Important to Learning in Mathematics

Writing to enhance learning is not a novel concept, the specific role of writing and how educators can maximize its benefits have been the focus of research on the topic over the past two decades (Prain & Hand, 2016). Prain and Hand (2016) concluded that writing represents a significant tool for learning across discipline areas. As such, writing to learn has been explored in terms of its efficaciousness in a variety of content areas. Klein and Boscolo (2016) noted that research has supported a shift from "discipline-neutral content literacy practices" (p. 324) to "discipline-specific approaches" (p. 324) in terms of writing to learn. Indeed, these researchers pointed to the work of Siebert and Draper (2008) who found that math literacy resources lacked crucial elements related to mathematics-specific learning revealing that discipline specialists are often not involved in the development of such materials. Powell and Hebert (2016) showed that general writing ability (including organization, theme development, and quantity) and computation skills were highly related to MW. Similarly, Hebert and Powell (2016) examined student characteristics of writing organization, mathematics vocabulary use, and representations in their MW. Their findings indicated that student organizational features in general writing and MW differed and their use of mathematics vocabulary and representations varied in the levels of performance.

Writing is associated with many benefits that are pertinent to student learning in mathematics including: (a) active learning, (b) self-reflection of learning elucidating both what is understood and what remains unclear, and (c) formative and summative assessment (Pugalee, 2005). Writing in mathematics allows the learner to organize, clarify, and reflect on ideas (Burns, 2004) and improve vocabulary use, sentence structure, and appropriate linking words (Cohen, Casa, Miller, & Firmender, 2015). Pugalee (2005) described writing as a significant tool for learning and suggested that it can be used to develop a student's capacity to reason mathematically. Similarly, Gillespie, Graham, Kiuhara, and Hebert (2014) argued that writing-to-learn in content areas such as math enhances student learning by helping students "comprehend, think critically, and construct new understandings about what they are learning" (p. 1044).

In this view, written communication is seen as essential to mathematical literacy where writing allows learners to make connections between content and process (Pugalee, 2005). Research indicates that writing in mathematics serves to develop a deeper mastery of the topic because it evokes critical thinking in the learner (Pugalee, 2005). More specifically, Pugalee (2005) expressed that writing is a conduit through which students can think about and communicate pertinent mathematical concepts. Accordingly, writing permits students to become empowered in terms of their personal mastery of the subject (Pugalee, 2005).

There is a growing body of empirical evidence supporting the notion that engaging in MW promotes positive academic outcomes in mathematics related to conceptual understanding and reasoning of students with and without disabilities across grade levels (Powell, Hebert, Cohen, Casa, & Firmender, 2017). Collectively, research supports that students who systematically engaged in MW interventions outperformed peers in control groups on mathematical-achievement assessments (e.g., Cohen et al., 2015; Cross, 2009; Jurdak & Abu Zein; 1998; Tan & Garces-Bascal, 2013), displayed better reasoning (Baxter, Woodward, & Olson, 2005; Cohen et al., 2015), exhibited improved problem solving (Fortescue, 1994; Moran, Swanson, Gerber, & Fung, 2014), demonstrated increased conceptual understanding (Fortescue, 1994; Martin, 2015), and fostered positive attitudes towards MW (Baxter et al., 2005).

Challenges for Students with Learning Disabilities

Many of the aforementioned studies evaluated performance of typical learners. For students with learning differences, such as those recognized as LD, research is limited. Students with LD experience different challenges to learning as an impact of their exceptionalities. As a combination of mathematics and writing, it renders a double burden to students, requiring both mathematics and writing skills (Powell & Hebert, 2016) in addition to potential difficulties with reading and challenges posed to processing. We recognize that MW challenges may not arise exclusively to students with *only* mathematics difficulties or reading and writing difficulties. Similarly, potential for differences in classifying and diagnosing LD may influence primary diagnosis or recognition of LD. Therefore, we included students for whom their learning differences were associated with a disability and it was recognized that these learning differences impacted performance in reading, writing, and/or mathematics. The primary diagnoses of participants varied and are shared with data in Table 4. All of the educational needs for the students were met in inclusive settings.

Firstly, the writing of expository texts, including informational and argumentative, represents a more demanding cognitive task than the composition of narrative texts for younger writers (Olive, Favart, Beauvais, & Beauvais, 2009). Students with LD, in particular, showed more challenges in expository writing, compared to typical achievers. The recent meta-analytic review by Graham, Collins, and Rigby-Wills (2017) well-documented writing characteristics of students with LD, such as difficulties in self-regulation, writing skills, motivation, and knowledge about writing (e.g., genre elements, vocabulary). Considering that expository writing requires knowledge on both general writing skills (e.g., conventions, mechanics) and genre-specific writing structures, it is not surprising that their expository essays usually include irrelevant, redundant, and erroneous texts (Strum & Rankin-Erickson, 2002).

Table 1. ***Means and Standard Deviations for Writing and Mathematics Scores of Participants***

	Groups	Numeracy Writing Speed (DWPM) *M* (*SD*)	General Writing prompt		Math Fact Fluency	
			TWW *M* (*SD*)	WSC *M* (*SD*)	DPM *M* (*SD*)	CPM *M* (*SD*)
All Participants	Treatment (*n* = 18)	70.18 (22.58)	73.85 (26.94)	71.44 (27.02)	22.33 (11.00)	30.43 (14.47)
	Control (*n* = 9)	63.89 (25.06)	89.66 (21.94)	88.00 (24.50)	27.33 (14.48)	36.22 (19.14)
Students with exceptionalities	Treatment (*n* = 4)	63.67 (7.15)	65.67 (26.94)	65.00 (29.61)	26.00 (11.90)	37.67 (15.40)
	Control (*n* = 3)	52.33 (6.55)	70.67 (19.14)	66.67 (21.06)	19.66 (9.42)	26.67 (12.50)

Note. DWPM = Digits written per minute; TWW = Total words written; WSC = Words spelled correctly; DPM = Correct digits per minute; CPM = Correct computations per minute.

Another challenge comes from barriers students encounter pertaining to algebraic thinking. Since algebraic thinking includes abstract concepts of numbers, operations, and symbols, students with LD experience difficulties in understanding and transforming those complex concepts to be represented in a concrete manner (Witzel, Mercer, & Miller, 2003) subsequently expressing their algebraic thinking process by using mathematical language (Hughes, Powell, & Stevens, 2016). Differences in mathematical understanding and fluency often lead students to experience difficulty describing their reasoning process in a cohesive and coherent manner (Baxter et al., 2005; Cross, 2009). Furthermore, students often feel challenged when they encounter an open-ended question because it requires conceptual understanding, application skills, vocabulary, syntactical knowledge, as well as mathematics facts fluency (Hughes et al., 2016; Jitendra, Sczesniak, & Deatline-Buchman, 2005). In addition, insufficient mathematical language ability can pose as a barrier to MW. Textbook glossaries include copious amounts of terms students are expected to know and apply

(Hughes, Powell, & Lee, 2018) and the words students encounter in mathematics have technical meanings with mathematical specificity (e.g., numerator, vertex; Hughes et al., 2016). Given these potential challenges to effectively engage students in MW, it is important that students have a clear strategy to develop MW skills anchored in both mathematics and writing research. A few studies documented the effects of MW interventions to promote algebraic thinking and problem solving of students with mathematical difficulties (e.g., Baxter et al., 2005; Case et al., 1992; Moran et al., 2014). Our study extended their efforts to build empirical evidence of MW intervention by developing a specific learning strategy to help them understand problem contexts, represent abstract algebraic concepts, visualize problem solving procedures, write about algebraic thinking and reasoning, and self-regulate this entire process.

CURRENT STUDY

Mathematical Writing Framework

Our intervention is situated in Bandura's (1986) social learning theory and complimented by McCutchen's (2011) model describing transition from novice to expert writers. The MW strategy follows the key tenets of social learning theory that learning is a cognitive process where learning occurs through observation in a social learning environment. The triad of behavior, personal and cognitive factors, and environment all influence the learner (Bandura, 1986). This theory explains the importance of modeling desired behaviors by demonstrating how to complete each step in the process. It also accounts for personal factors, such as learner motivation and self-efficacy, which are addressed in initial purpose and goal setting, and values feedback in social learning situations.

McCutchen (2011) proposed a theoretical model to describe how students transition from novice to expert writers. In this model, two components are pivotal to writing quality: (a) fluent language generation processes, and (b) extensive knowledge relevant to writing. The former includes what McCutchen described as a combination of the development of transcription fluency (e.g. spelling and handwriting) and text generation fluency (i.e. idea development to written expression). The latter represents the combination of both an understanding of discourse forms as well as comprehensive subject-matter knowledge. McCutchen explained the interconnection of language, knowledge, and memory processes where the mastery of the first two factors enables the writer to avoid constraints related to short-term working memory and capitalize on long-term working memory mechanisms. The SRSD strategy was designed to address these requirements and thus move the novice mathematical writer toward a level of expertise by providing a framework that supports the interconnection of language and subject-specific knowledge in the production of MW.

The Intervention

Instruction is a moderating variable on the effects of writing to learn (Graham & Hebert, 2011); consequently, just providing MW opportunities may not be as advantageous as teaching students how to strategically MW. Given that students who struggle to write effectively are less able to use this medium to enhance their learning in the content areas (Graham & Perin, 2007), interventions that foreground explicit writing instruction within the content areas may have a greater impact on student learning. We extended this concept to MW by teaching students a MW strategy anchored in evidence-based practices for students with disabilities. These include self-regulated strategy development (SRSD) to address writing, heuristics to address comprehension of mathematics word problems, and visual representations (VR) to support understanding of mathematical concepts.

SRSD. SRSD is an evidence-based practices in teaching writing for students with and without disabilities (Harris, Graham, Aitken, Barkel, Houson, & Ray, 2017). SRSD teaches writing and self-regulation strategies that help students manage the writing process including planning, writing, editing, and revising through explicit and purposeful instruction (e.g., Harris & Graham, 1999). The multifaceted nature of SRSD instruction provides a systematic approach to actively engage students in rich discourse regarding writing and writing behaviors and provide students with knowledge and skills to employ a strategic approach to content-specific writing (e.g., Harris & Graham, 2017; Harris, Ray, Graham, & Houston, 2019). Empowering aspects of SRSD include attention to motivation, self-efficacy, self-assessment, and attributions of success to efforts (e.g., Harris & Graham, 2017). Because many students with disabilities experience failures in writing due to cognitive, affective, and behavioral challenges when they write, SRSD combines structured, individualized instruction, and self-regulation procedures with strategy instruction to meet students' needs (Berry & Mason, 2012). Studies examining the effects of SRSD demonstrated greater impacts on students' writing quality, quantity, knowledge, and strategy use compared to writing instruction that did not use SRSD (Gillespie & Graham, 2014). SRSD writing strategies have been effectively used with informational texts (Ennis, 2016), specifically with social studies content (Mason, Snyder, Sukhram, & Kedem, 2006), history content (De La Paz & Felton, 2010; Wissinger & De La Paz, 2016), and science (Mason et al., 2006).

Researchers have extended SRSD interventions to mathematical performance, such as multi-step equations (Cuenca-Carlino, Freeman-Green, Stephenson, & Hauth, 2016) and addition and subtraction word problems (Case, Harris, & Graham, 1992; Cassel & Reid, 1996), for students with learning difficulties and disabilities, but these interventions focused on computations and not written expression. Additionally, strategy instruction was shown to improve accuracy for learners in terms of word-problem solving (Baker, Gersten, & Lee, 2002). Because SRSD research consistently yields positive results to support higher-level cognitive processes involved in learning a target behavior, including self-regulation components for using the strategy effectively, it is expected to be a potentially effective instructional approach for promoting student MW and mathematical reasoning (Kiuhara & Witzel, 2014).

Heuristics. Heuristics, or a heuristic approach, in mathematics emphasizes general strategies and procedures for mathematical-problem solving. Students are taught these strategies and procedures to best situate them to adequately devise a plan to solve a problem and follow the plan to fruition, albeit the specific steps may differ by approach (e.g., Case et al., 1992; Jitendra, Star, Dupuis, & Rodriguez, 2013; Jitendra et al., 2009; Polya, 1990). Incorporating a heuristic approach to problem solving addresses potential differences in cognitive and meta-cognitive skills for students with LD (Montague, Enders, & Dietz, 2011).

VR. Many students find the abstraction of mathematics, especially for concepts encountered in middle grades, challenging. VRs (e.g., drawing, picture, diagrams) may be used to help students make sense of abstract concepts, as the VR provides a visual depiction of the abstract idea (van Garderen & Montague, 2003). The National Mathematics Advisory Panel (2008) reported that VRs effectively support students with and without disabilities. VRs have longstanding evidence supporting effectiveness specifically supporting students with LD, so much so that Jitendra, Nelson, Pulles, Kiss, and Houseworth (2016) share that VRs meet stringent criteria as an evidence-based practice for students with LD. For this reason, we purposefully incorporated VRs in the MW strategy as an effort to build conceptual understanding and communicate these concepts via written expression.

Research Questions

Empirical evidence is needed on how to support MW instruction and increase student achievement, warranting this exploratory pilot study to evaluate MW and the effects of a MW strategy intervention on student performance and serve as a proof-of-concept to extend SRSD to MW. Consequently, in this study we investigated the relationships between mathematics, writing performance, and MW. We also evaluated academic outcomes of a MW strategy intervention. We proposed three research questions to address these explorations:

1. What are the relationships between MW and any other variables related to writing and mathematical performance?
2. To explore the potential predictors for MW, we asked: Which variables can be possible predictors of MW?
3. What are the effects of a MW strategy intervention on student MW outcomes?

METHOD

Setting and Participants

The proof-of-concept pilot study took place in a small tuition-free, public charter middle school serving students in Grades 5 though 8 in a midwestern state. Approximately 20% of the student population is identified as being eligible for special education services. The majority of students identified as Caucasian. The intervention took place during the designated writing class, which occurred three afternoons a week. Writing classes were inclusive and mixed (i.e., fifth and sixth graders) classes. Twenty-seven students consented to participate, which included seven students with exceptionalities (Attention deficit hyperactivity disorder [ADHD], *n* = 3; autism spectrum disorder [ASD], *n* = 2; speech and language [SPL], *n* = 1; Inactive individualized education program [IEP], *n* = 1), who received instruction in an inclusive setting. The classes were randomly assigned to treatment or control conditions. Specifically, there were 10 male and 8 female students assigned to the treatment group and 3 male and 6 female students in the control group. Table 1 displays their writing and mathematics fluency scores before the intervention. The treatment group received the MW intervention and the control group received business-as-usual instruction and practice on expository/informational writing.

Mathematical Writing Strategy Intervention

The intervention consisted of six, 40-50 minute lessons that were taught by the first and third authors. The number of sessions and instructional time were determined after discussion with teachers on the feasibility of the intervention, considering the instructional procedures of SRSD (Harris, Graham, Mason, & Friedlander, 2008). All lessons were taught to the whole class. Lessons were taught using SRSD framework described by Harris and colleagues (2008) and anchored in explicit instruction, where the instructor (a) developed students' background knowledge on MW; (b) discussed what the strategy is and why it is important to use; (c) modeled new information, utilizing metacognitive talk to model cognitive process through tasks, (d) provided activities to memorize the strategy, and (e) engaged students in guided practice, and (f) culminated in monitored independent practice. There were six major components of the MW strategy modeled during the intervention, (a) making sense of the word problem, (b) determining an appropriate plan to solve the problem, (c) drawing a representation of the problem, (d) explaining problem solving and reasoning, (e) concluding the paragraph by stating the answer, and (f) systematically checking all components of work.

In the first lessons, the first author started with a discussion question on when to communicate mathematically to activate students' prior knowledge. Then, introduced the purpose for MW and discussed with students what makes it "good," with examples and nonexamples. Students were then introduced to the writing strategy to help them write mathematically. In the second lessons, students were taught how to apply the first two components of learning strategies (i.e., making sense of word problem; determining an appropriate plan to solve the problem) to their MW. From this lesson, students learned how to use self-statement for their goal setting, self-instruction, self-monitoring, and self-reinforcement (Harris et al., 2008). In the third lessons, students reviewed transfer skills, discussed math writing goals, and learned to do the next three components of learning strategies (i.e., drawing a representation of the problem; explaining problem solving and reasoning; concluding the paragraph by stating the answer). The instructors modeled all the steps of learning strategies and provided guided practice with partially-worked examples.

In the fourth lesson, students learned to incorporate the first five steps of the problem and check their work (i.e., sixth component of learning strategies). The instructor modeled how to check work by retrieving the previously learned components and evaluating the quality of math writing. Students used colored stickers to check if they addressed all components of the learning strategies. During the fifth and final lessons, students evaluated their math writing from the previous lesson themselves and continued to work on MW by integrating all components of learning strategies independently. Students worked with their partner from the previous lessons to provide formative peer-feedback. In the final lesson, students began to generalize learned information by practicing MW on word problems that looked similar to what they would encounter on a classroom math assignment.

At the beginning of each lesson, students received a writing folder that contained all of the materials for the day. Teachers' scripts and Power Point materials were developed prior to the intervention. All of the lessons were delivered by the first author, a former fourth-grade teacher and current assistant professor in special education, and the third author, a 14-year veteran teacher and current doctoral student in special education. All lessons were audio-recorded for fidelity purposes only. The second author compared students' materials and audio tapes to the scripted lessons and used a checklist to determine if lessons followed instructional procedures. Comparison resulted in full agreement of 100% fidelity of treatment.

Variables and Measurements

Numerical digit writing speed. Students were given 1 minute to write the digits 0-9 as many times as possible. Assessments were scored for digits written per minute (DWPM). For the interrater agreement, a graduate student who was not related to this study counted the number of digits that the respondents wrote after the second author scored. It resulted in 100% agreement.

General writing prompt. Students were given a writing prompt (Harris et al. 2008), and 1 minute to plan and 6 minutes to write. Assessments were scored for total words written (TWW) and words spelled correctly (WSC). To determine the interrater agreement, the second author and the aforementioned graduate student scored the TWW and WSC separately and compared results, which resulted in 92.3% of agreement for TWW and 99.8% for WSC.

Mathematics fact fluency. Students were given 1 minute to complete a mathematics fact fluency page with simple, mixed facts for addition, subtraction, multiplication, and division. Assessments were scored for correct digits per minute (DPM) and number of correct answers on computations per minute (CPM). The second author and the graduate student separately scored the fact fluency measure for DPM and CPM and compared results to determine agreement. This resulted in 100% of agreement for DPM and 99.8% for CPM.

Mathematical writing. Students were given 30 minutes to complete three mathematics problems that resembled end-of-year State open-response written expression assessment questions and one short open-response question and 10 additional minutes to complete an open-response problem. The quality of students' responses was scored by the second author, using a four-point holistic rubric adjusted from the Pennsylvania System of School Assessment (PSSA) for fifth grade (Pennsylvania Department of Education Bureau of Curriculum, Assessment and Instruction, 2016). In the rubric, a student that demonstrated *thorough* understanding receives a score of 4, *general* understanding receives a score of 3, *partial* understanding receives a score of 2, *minimal* understanding receives a score of 1, and no response receives a score of 0. We aligned our scoring to mirror the scoring system with student samples from the guidance of the State assessment. Considering accuracy of answer, if student responses included thorough explanation on their reasoning, but the answer was incorrect due to minimal computation error, we scored it as 4. If the answer was correct without any explanation on their reasoning procedures, they received a score of 1. All data were collected by the first author and shared with the scorers as one lot that included pre and post data from the treatment and comparison groups. This was done to minimize reviewer bias. One-third of data were randomly selected and coded by the third author who was trained to score the quality of student responses to determine interrater agreement. It resulted in 83.3 % of agreement. The second and third authors discussed the disagreements in scoring to reach 100% agreement prior to data analyses.

Data Analyses

There were a few missing data points during the data collection. The total portion of attrition was 5.4% and average 6.4% across the variables, ranging from 0 to 11.1%. They were transformed to the means for each variable and were analyzed to preserve important characteristics of the data set, using SPSS Version 25. Three statistical analyses were implemented to address research questions. To explore the relationships among the variables, Pearson's *r* was calculated to identify the correlations among the variables, and a multiple linear regression was used to determine the predictors of MW. To evaluate the effectiveness of the MW intervention, an independent samples t-test was conducted.

RESULTS

In this pilot study, we asked three research questions: (a) What are the relationships between MW and any other variables related to writing and mathematical performance? (b) Which variables can be possible predictors of MW? (c) What are the effects of a MW strategy intervention on student MW outcomes? The results from the research questions are shared in order of their presentation.

Correlations

Our first research question inquired about the relationships between MW and other variables related to writing and mathematical performance. We ran correlational statistics to evaluate mutual relationships between performance factors for numerical digit writing speed (i.e., DWPM), general writing prompt scores (i.e., TWW, WSC), mathematics fact fluency (i.e., DPM, CPM), and MW scores. Performance on MW had significant correlations with mathematics fact fluency for DPM ($r = .589$, $p = .001$) and mathematics fact fluency for CPM ($r = .634$, $p < .001$). MW scores did not have correlations with any other variables (i.e., DWPM, WSC, TWW). Numerical writing speed did not yield any significant correlation with other variables. See Table 2 for more details.

Table 2. ***Correlations Between Variables***

Measures	1	2	3	4	5	6
1. Numerical Digit Writing Speed (DWPM)	—					
2. General Writing Prompt (TWW)	.047	—				
3. General Writing Prompt (WSC)	.092	.995**	—			
4. Math Fact Fluency (CPM)	.195	.571**	.593**	—		
5. Math Fact Fluency (DPM)	.209	.558**	.580**	.994**	—	
6. Mathematical writing	-.050	.298	.284	.589**	.634**	—

Note. *Note*. DWPM = Digits written per minute; TWW = Total words written; WSC = Words spelled correctly; DPM = Correct digits per minute; CPM = Correct computations per minute; Correlations with ** are significant at $p < .001$.

Table 3. ***Results of Multiple Regression Analysis with Mathematical Writing as a Dependent Variable***

Independent Variable	Unstandardized Coefficients		Standardized Coefficients	*t*	*P*	sr^2
	B	*SE*	*ß*			
Numeracy Writing Speed (DWPM)	-.016	.017	-.1330	-..918	.369	-.197
General Writing Prompt (TWW)	.282	.157	2.635	1.798	.087	.365
General Writing Prompt (WSC)	.280	.154	2.707	1.817	.083	.369
Math Fact Fluency (CPM)	.856	.279	3.750	3.063	.006	.556
Math Fact Fluency (DPM)	.781	.211	4.490	3.700	.001	.628

Note. DWPM = Digits written per minute; TWW = Total words written; WSC = Words spelled correctly; DPM = Correct digits per minute; CPM = Correct computations per minute.

Potential Predictors of Mathematical Writing

We conducted a multiple regression analysis to identify the predictors for students' quality of MW. Prior to the multiple regression, we conducted preliminary analyses for data screening such as homogeneity of variance, homoscedasticity, independent errors, and collinearity between predictors. There were no significant problems in the data distributions. The regression model was obtained with $F(5, 21) =$

8.109, $p < .001$, indicating that the linear combination of predictors was significantly related to MW outcomes.

Table 3 displays the results of multiple regression analysis for MW quality as a dependent variable. Among five independent variables, two variables related to mathematics fact fluency (i.e., DPM and CPM) were significant predictors for MW. Squared semi-partial correlations indicated that DPM accounted for 55.6% of unique variance and CPM explained 62.8% of unique variance. DWPM, TWW, and WSC did not contribute to explaining the variance in MW quality.

Effects of Mathematical Writing Intervention

We conducted independent and dependent two samples t-tests to examine the effectiveness of the MW intervention. Prior to the intervention, to ensure there were no significant differences at the beginning of the study, independent t-tests were conducted to evaluate differences between groups in writing speed, general writing performance, and mathematics fact fluency. T-test indicated that at the 0.05 level of significance there were no significant differences in pretests between groups on DWPM ($t = .569$, $p = .574$), TWW ($t = 1.523$, $p = .140$), WSC ($t = 1.546$, $p = .135$), DPM ($t = .881$, $p = .387$), CPM ($t = 1.002$, $p = .326$), and pretest scores for MW ($t = .764$, $p = .452$). There was an increase in mean scores from pre to post measures for the treatment group, however, differences were not statistically significant. Similarly, a decrease in mean scores was detected for the control group; however, there were no statistically significant differences from pre to post measures. There was no significant difference on post-test measures and no interaction effect. After intervention, the result indicated that there was no significant difference between groups in the posttest for MW ($t = .427$, $p = .673$) despite change in performance difference.

Table 4. ***Descriptive Statistics for Mathematical Writing***

	Groups	N	Exceptionality (*n*)	Pretest *M* (*SD*)	Posttest *M* (*SD*)
All Students	Treatment	18	-	3.08 (2.58)	4.53 (3.16)
	Control	9	-	4.11 (3.22)	3.84 (2.60)
Students with exceptionalities	Treatment	4	ASD = 2; ADHD = 1; SPL = 1	4.78 (3.11)	5.64 (3.14)
	Control	3	ADHD = 2; Other =1	4.33 (3.30)	3.33 (2.05)

Finally, we disaggregated data for students with disabilities. Means and standard deviations for pre and post assessments are available in Table 4. MW scores for students with exceptionalities in the control group decreased 1.45 points from pre to post assessment, while scores for student with exceptionalities in the treatment group increased an average of 0.86 points. Differences in scores from pre to post assessments were not significant.

DISCUSSION

The findings shared here are the result of a pilot study to test a proof-of-concept writing strategy specially designed to support students' MW. To our knowledge this is the first study that not only specifically targets a MW intervention, but also adds to a growing body of research that evaluates SRSD to support mathematical achievement (e.g., Cuenca-Carlino et al., 2016) and an established body of research supporting SRSD to support writing achievement (Graham & Harris, 2003).

Mathematical Writing Relationships

Mathematics fact fluency, identified as DPM and CPM, correlated with MW quality. While the DPM had a slightly greater correlation to MW outcomes, the differences in the two scores were negligible and the ease of score problems correct rather than digits correct favor scoring problems correct. Our findings suggest that mathematics fact fluency was a better indicator of MW performance. This supports initial findings from Powell and Hebert (2016), who also documented relationship between mathematics facts fluency and writing performance. This finding also supports the conclusions of McCutchen (1986) related to the positive relationship between content knowledge and coherent texts in a study conducted with students in the fourth, sixth, and eighth grades.

Writing speed, as assessed by DWPM, did not correlate with any of the other outcomes, suggesting that the motor skills and speed of writing did not relate to quality of outcomes. This finding differs from the conclusions of Levy and Ransdel (1995) who reported a positive correlation between text quality and transcription fluency. However, it should also be noted that Berninger and Swanson (1994) posited that student age may represent an important factor in the amount of unique variance accounted for by transcription fluency for primary grade children versus older children, such as participants in this study. Additionally, digit writing fluency requires students to write quickly, but does not require students to process mathematics, like mathematics fact fluency, which had a high correlation. Our small sample size precludes us from overgeneralizing, but the initial findings are worth future analyses and align with theories of memory as proposed by McCutchen (2011). MW requires students to activate short-term and long-term memory to solve mathematics problems (Mabbott & Bisanz, 2008) and explain reasoning for these problems via written expression (Hebert & Powell, 2016).

Potential Predictors for Mathematical Writing

The multiple linear regression model for MW quality showed potential meaningful implications for MW. Results indicated that robust mathematics fact fluency had a strong co-occurrence with quality MW, accounting for a large percentage of variation for MW quality. This supports evidence from the field that mathematics fact fluency is one of the critical components for overall mathematics outcomes (NCTM, 2010). Our findings do not suggest causality (e.g., that fact fluency interventions will directly increase MW performance), it does draw attention to importance of basic facts knowledge for broad mathematics applications and expressions. Several researchers have demonstrated the important relationship between fact fluency and mathematical comprehension as well as that of high-order thinking ability such as mathematical reasoning (Calhoon, Emerson, Flores, & Houchins, 2007; Gerber & Semmel, 1994). Because MW requires students to generate accurate reasoning processes within a limited time, automaticity of mathematics facts may allow students to focus cognitive energy on articulating mathematical reasoning and organizing written expressions.

Effects of a Mathematical Writing Intervention

Our third research question evaluated the impact of a MW intervention on MW outcomes. The findings are promising, as students demonstrated improvement in MW performance after only six sessions. Despite the significant increase in performance for students in the intervention group and the lack of increase in student performance for students in the control group, the difference in change was not significant enough to establish an interaction effect, which was likely impacted by a small sample size and limited dosage of the intervention. Our findings support proof-of-concept and warrant further investigation with more participants and for a longer period of time.

Observational analysis of content shared some evidence for students' improvements in mathematical reasoning process from their responses, but their writing still showed limited mathematical vocabulary use (e.g., *simplify*, *term*, *greatest*, etc.). Even when the representations (e.g., equations, diagrams) and answers were correct, insufficient capability to use mathematical vocabulary seemed to hinder quality writing of precision in MW. McCutchen (2011) points out that novice learners, especially those with working memory constraints, may not be able to retrieve essential vocabulary or have not yet learned these terms and committed them to memory. It indicates that explicit instruction for mathematical language should also be provided, along with MW instruction (Hughes et al., 2016). Future analyses may include systematic and discourse analysis of writing.

Limitations

We acknowledge a few noteworthy limitations to the study. First, we piloted this intervention with a small sample size. We believe the sample size was the greatest limitation and may have contributed to the inability to detect effect size difference between the performance of the two groups. Despite our small *N* as a limitation, we still detected interesting changes worth future exploration.

Second, we worked with the teachers in the spring semester to complete the intervention prior to mandatory state assessments. Due to scheduling and snow days, we were required to complete the intervention in six sessions. While it was encouraging that there were measurable changes from pre to post assessments in this short time, students did not practice writing until the third of six lessons. Given this short period of practice time, we anticipate that increased dosage of the intervention and opportunity to apply the strategy will increase student performance; however future evaluation is needed to support or refute this hypothesis.

We used both researcher-created assessments and normed assessments; however, future research is needed to evaluate if the strategy use translates to improved performance on state assessment measures of mathematical reasoning. We used grade-level mathematics word problems, which demonstrated to be too difficult for some students when trying to simultaneously engage in the writing process. In the future, we recommend allowing students to first practice MW with mathematics standards that they have already mastered, which will allow them to focus on the written expression portion of MW, with acceleration to grade-level or personally-appropriate mathematics standards.

Implications for Research and Practice

We recognize several implications for research. As educators in K-12 settings continue to provide multiple modes for students to develop and express mathematical reasoning (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010), including written expression of mathematics (NCTM, 2000), it is increasingly important to support students with exceptionalities to develop strategies for success. More research is needed in the field to evaluate how students structure mathematical written expression and argumentation and how instruction can support student success.

MW spans the realms of language arts (e.g., reading, writing) and mathematics. It is important to explore how skills from both areas uniquely contribute to quality MW and specific ways to balance the support in language (e.g., writing structure, language) and mathematics (e.g., conceptual understanding). Given heterogenous mathematical challenges for students with and without exceptionalities (e.g., working memory, content-specific skills, language; Jordan, Hansen, Fuchs, Siegler, Gersten, & Micklos, 2013) greater exploration regarding contributing factors to success is warranted, especially situated in how to best help students with varying academic and behavioral challenges.

There are several ways in which these findings can inform practice. First, we hope to draw attention to the unique (matrriage) of writing and mathematics required for MW, and draw attention to how differences in learning for students with LD may impact MW outcomes. Second, our findings suggest that students benefit from intervention designed to support MW and that expository writing instruction may not be enough to support content-specific writing in mathematics, such as on open-response word problems. Instead, teachers may need to purposefully consider how both mathematics and writing demand contribute to outcomes. Finally, we shared that SRSD can be extended to MW to support student outcomes. Students may benefit from an SRSD MW intervention.

Conclusion

Writing in specific content areas is different from general writing because it requires general understanding for genre- or domain-specific writing (Hebert, Bohaty, Nelson, & Roehling, 2018; Hebert & Powell, 2016). Researchers in systemic functional linguistics have identified specialized linguistic features of the mathematics register (e.g. multiple semiotics systems and grammar patterns) which significantly distinguish oral and written mathematical outputs from everyday language (Schleppegrell, 2007). Like other subject areas such as social studies, history, and science, MW also needs explicit instructional supports for mathematical language (Hughes et al., 2016), text structures and organization in informal or formal ways (Powell & Hebert, 2016), and VR (Hebert & Powell, 2016).

We view the findings from this pilot study as promising and moving the fields of writing and mathematics forward in a positive direction. While more research is needed to evaluate written expression of mathematical reasoning, especially considering dosage and variety of writing and mathematical outcomes, we provide preliminary evidence that a systematic MW intervention can increase students' MW performance.

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